On full-separating sets in graphs - talk

Dipayan Chakraborty, LIMOS, Université Clermont-Auvergne, France, dipayan.chakraborty@uca.fr
Annegret K. Wagler, LIMOS, Université Clermont-Auvergne, France, annegret.wagler@uca.fr

Identification problems in graphs model the monitoring and detection of faults in a network of nodes by usually selecting a vertex subset (the set of "detectors") that possesses both a *dominating* and a *separating* property. Such a vertex subset is often called a *code* of the graph. Given a graph $G = (V, E)$, we denote by $N(v)$ and $N[v]$ the *open* and *closed neighborhood*, respectively, of a vertex $v \in V$. Then, a set $C \subseteq V$ is called *closed-separating* (respectively, *open-separating*) if $N[u] \cap C \neq N[v] \cap C$ (respectively, $N(u) \cap C \neq N(v) \cap C$) for any two distinct vertices $u, v \in V$. In addition, C is *locating* if $N(u) \cap C \neq N(v) \cap C$ for all distinct $u, v \in V \setminus C$.

Combining these three separation properties with those of *domination* and *total-domination*, the following six codes have been studied so far in the literature : *locating dominating codes* [4], *locating total-dominating codes* [2], *identifying codes* [3], *identifying total-dominating codes* [2], *open-separating dominating codes* [1] and *open neighborhood locating-dominating sets* [5]. Location is useful when the detectors do not go faulty over time. Closed separation can withstand faults whereby a detector looses the ability to distinguish whether a fault has occured at itself or at another node. Open separation is helpful in the functionning of a network when a fault destroys a detector.

In this work, we ask, what happens if both the above fault types occur simultaneously in a network ? This makes us introduce a stronger form of separating sets that are both open- and closed-separating. We call such a set to be a *full-separating set* ; and combining this with both domination and total-domination, we get *full-separating dominating codes* (or FD-codes) and *full-separating total-dominating codes* (or FTD-codes). We denote the minimum order of these two new codes of a graph G by $\gamma^{\mathrm{FD}}(G)$ and $\gamma^{\mathrm{FTD}}(G)$, respectively. Like it is the case for the other six codes, we show that the problems of finding a minimum FD- and FTD-code are NP-complete.

Interestingly, we also find that $\gamma^{\mathrm{FTD}}(.)$ and $\gamma^{\mathrm{FD}}(.)$ differ by at most 1. However, we show that, despite this small difference, it is NP-hard to decide in general if the two code numbers differ or equal each other on a given graph. Apart from the complexity results, we also investigate the two code numbers on some well-known graph classes like paths, cycles, half-graphs and headless spiders and find exactly the values of these parameters on such graphs.

Références

[1] Chakraborty, D., Wagler, A.K. : Open-Separating Dominating Codes in Graphs. In : Basu, A., Mahjoub, A.R., Salazar González, J.J. (eds) Combinatorial Optimization. ISCO 2024, Lecture Notes in Computer Science **14594**, pp. 137–151 (2024).

[2] Haynes, T.W., Henning, M. A., Howard, J. : Locating and total-dominating sets in trees, Discrete Applied Mathematics **154**, pp. 1293–1300 (2006).

[3] Karpovsky, M.G., Chakrabarty, K., Levitin, L.B. : On a new class of codes for identifying vertices in graphs. IEEE Transactions on Information Theory **44**(2), pp. 599–611 (1998).

[4] Slater, P.J. : Dominating and reference sets in a graph, Journal of Mathematical and Physical Sciences **22**, pp. 445–455 (1988).

[5] Seo, S.J., Slater, P.J. : Open neighborhood locating dominating sets, Australasian Journal of Combinatorics **46**, pp. 109–119 (2010).