Constructing tree-based linear oblivious routings

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Given a graph $G = (V, E)$ with n vertices and m edges, and a demand vector on the *pair* of vertices, we would like to compute a flow that satisfies the demand. A routing is an operator that outputs a flow satisfying an input demand. In this presentation, we focus on *linear oblivious routings* (LOR). In a LOR each pair of vertices has an associated pre-computed flow. Whenever one asks for a demand d between vertices s and t, we multiply the associated pre-computed flow by d to obtain the output flow. For a demand, the flow obtained can be seen as a vector on the edges. Given a cost function on the edges, we define the competitive-ratio of a routing $\mathcal{A}$ as :

$$\text{Competitive-ratio}(\mathcal{A}) := \max_{\text{demand } \mathbf{d}} \frac{\text{cost}(\mathcal{A}(\mathbf{d}))}{\text{cost}(\mathcal{OPT}(\mathbf{d}))}.$$

In order to construct a linear oblivious routing, Räcke introduced in [1] *tree-based LOR*. The motivation behind this construction is that in a tree, there is only one way to route the demand between any two vertices. Formally, a *tree-based LOR* is a collection of spanning trees, where for any demand vector, we route a fraction of the demand along each spanning tree. $\mathcal{O}(\log m)$-competitive tree-based LOR exist on any non-oriented graphs when the cost function is $||.||_p$ for $1 \leq p \leq \infty$ [2]. The current algorithm to construct $\mathcal{O}(\log m)$-competitive tree-based LOR is the following : Given a tree-based LOR, approximate the demand achieving its competitive ratio, then compute a good tree-based LOR for that specific demand and take a linear combination of both routings. The obtained results is a tree-based LOR with a slightly smaller competitive-ratio [3]. We developed a faster algorithm to approximate the demand achieving the competitive-ratio, saving a factor $\mathcal{O}(m^3)$ in the total running time.

Références

[1] H. Räcke, *Optimal Hierarchical Decompositions for Congestion Minimization in Networks*, STOC (2008)

[2] M. Englert and H. Räcke, *Oblivious Routing for the Lp-norm*, FOCS (2009), 32–40

[3] A. Bhaskara and A. Vijayaraghavan, *Approximating Matrix P-Norms*, Siam J. Discrete Algorithm **22** (2011), 497-511