Hypergraphs of linear systems over the two-element field and quantum contextuality proofs

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MaxLin2 is the problem of maximizing the number of satisfied equations in a linear system over the two-element field [2]. Hypergraphs can naturally be associated with these systems and the subsets of their equations satisfied and unsatisfied by a given zero-one assignment of their variables. They provide a useful visual perspective on these abstract algebraic problems.

After introducing the quantum phenomenon called *contextuality* (see, e. g., [1] for a comprehensive review), we link its existence proofs with MaxLin2. Then we present a new heuristic method for MaxLin2, inspired by the (hyper)graphic perspective. It brings out particular symmetrical hypergraphs, as detailed in a joint work with Frédéric Holweck [3].

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