Edge-recoloring with non-increasing potential

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Vizing's theorem states that the minimum number of colors required to properly color the edges of a graph, is either Δ or $\Delta + 1$. Specifically, Vizing proved that, from any proper k-edge-coloring with $k > \Delta + 1$, one can obtain a proper $(\Delta + 1)$-edge-coloring of a graph using Kempe changes. This result provides an algorithm to sample a proper k-edge-coloring of a graph for $k \geq \Delta + 1$. Thus, it naturally raises the question of whether there is an efficient procedure to uniformly sample proper k-edge-colorings, for $k \geq \Delta + 1$.

Recently, Wang, Zhang, and Zhang [1] showed that Glauber dynamics is an efficient uniform sampler for proper k-edge-colorings when $k \geq (2+o(1))\Delta$. However, their method is proven to be unable to break the 2Δ threshold.

In 2020, Dotan, Linial and Peled [2] proposed another algorithm to generate uniformly random proper $(2n-1)$-edge-colorings of the complete graph K_{2n}. A version of this algorithm consists in starting from an arbitrary edge-coloring (not necessarily proper) and successively recoloring randomly chosen edges if some predefined potential ϕ does not increase. Based on simulations, they conjectured that this random walk on the $(2n-1)$-edge-colourings of K_{2n} almost surely reaches a proper edge-coloring, and that it does so in $\tilde{O}(n^4)$ steps.

In this talk, we will prove that for $k \geq \Delta + 1$, there is always a reconfiguration sequence from any k-edge-coloring to some proper k-edge-coloring that does not increase the potential ϕ.

Références

[1] Yulin Wang, Chihao Zhang, and Zihan Zhang. *Sampling Proper Colorings on Line Graphs Using* $(1 + o(1))\Delta$ *Colors.* arXiv preprint arXiv :2307.08080, 2024.

[2] Dotan, M. and Linial, N. and Peled, Y., *Efficient, local and symmetric Markov chains that generate one-factorizations*, Acta Math. Hungar **161** (2020), 557–568.