Subcoloring of (unit) disks graphs

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A subcoloring of a graph is a partition of its vertex set such that the subgraph induced by each subset does not contain a path on three vertices as an induced subgraph (or equivalently, is a disjoint union of cliques). By extension, the subchromatic number of a graph is the minimum number of colors of a subcoloring of it, and the k-subcoloring problem asks whether a given graph has subchromatic number at most k. This problem has already been studied from algorithmic and structural aspects, in particular for several subclasses of perfect graphs or planar graphs. In this paper we focus on geometrical intersection graphs, and more precisely (unit) disk graphs. We first prove that 2-Subcoloring remains NP-hard in unit disk graphs with clique size at most 3. We then provide several algorithms and structural bounds. We show that the subchromatic number of a unit disk graph is at most 7, and can be 3-approximated in polynomial time. Finally, we show that every n-vertex disk graph admits a subcoloring with at most $O(\log^3(n))$ colors. This is achieved by defining a decomposition and special kind of co-comparability disk graphs that we call Δ-disk graphs.

Références

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