A simple quadratic kernel for Token Jumping on surfaces

Daniel W. Cranston, Department of Computer Science, Virginia Commonwealth University, dcranston@vcu.edu
Moritz Mühlenthaler, G-SCOP, Grenoble INP, moritz.muhlenthaler@grenoble-inp.fr
Benjamin Peyrille, G-SCOP, Univ. Grenoble Alpes, benjamin.peyrille@grenoble-inp.fr

The Independent Set Reconfiguration - Token Jumping (ISR-TJ) problem asks whether, given a graph G and two independent sets of *tokens* I and J of G, we can transform I into J by changing the position of a single token in each step and having an independent set of tokens throughout. We show that there is a polynomial-time algorithm that, given an instance of ISR-TJ, computes an equivalent instance of size $O(g^2 + gk + k^2)$, where g is the genus of the input graph and k is the size of the independent sets. Our algorithm is very simple and does not require an embedding of the input graph.

Our approach, is inspired by a technique introduced by Takehiro et al. [1] and used in subsequent articles. The main idea is to partition the graph into four sets : the vertices of $I \cup J$, the vertices $\mathcal{C}_1$ that at most one neighbor in $I \cup J$, the vertices $\mathcal{C}_2$ that have exactly two neighbors in $I \cup J$ and the vertices $\mathcal{C}_3$ that have at least three neighbors in $I \cup J$. We can easily bound the size of $\mathcal{C}_1$ and $\mathcal{C}_3$ using Heawood's number and Euler's formula. We then exploit the structure of any drawing of G to reduce the size of $\mathcal{C}_2$.

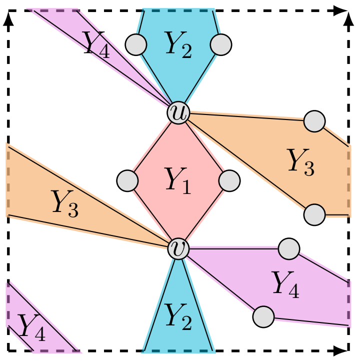

FIGURE 1 – Vertices of $\mathcal{C}_2$ adjacent to $Y = \{u, v\}$ on a torus ($g = 1$).

Références

[1] Takehiro Ito, Marcin Jakub Kaminski, and Hirotaka Ono. *Fixed-parameter tractability of token jumping on planar graphs*, ISAAC 2014.