Packing forests with constraints on the number of connected components

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The seminal papers of Edmonds [2], Nash-Williams [4] and Tutte [5] have laid the foundations of the theories of packing arborescences and packing trees. The directed version has been extensively investigated, resulting in a great number of generalizations. In contrast, the undirected version has been marginally considered. The aim of this paper is to further develop the theories of packing trees and forests. Our main result on graphs characterizes the existence of a packing of k forests, $F_1, \dots, F_k$, in a graph G such that each vertex of G belongs to exactly h of the forests, the number of connected components of each F_i is between $\ell(i)$ and $\ell'(i)$ and the total number of connected components in the packing is between α and β. Finally, we extend this result to hypergraphs and dypergraphs, the latter giving a generalization of a theorem of Bérczi and Frank [1]. As a matter of fact, this research was motivated by the paper of Bérczi and Frank [1].

Références

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