Coloring bridge-free antiprismatic graphs

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The vertex coloring problem is known to be NP-complete in the general case but it is in P if restricted to certain classes of graph. Those of greatest interest to us are the classes defined by forbidden induced subgraphs. Given a set of graphs $\mathcal{H}$, we denote Free $\mathcal{H}$ the class of graphs that does not contain any graph of $\mathcal{H}$ as induced subgraph. Among the classes for which the complexity of the coloring problem is still unknown, we focused on Free$\{K_{1,3}, 2P_1 + P_2, 4K_1\}$.

A graph G is *prismatic* if for every triangle T of G, every vertex of G that is not in T, has a unique neighbor in T. The complement of a prismatic graph is called antiprismatic. It can be easily seen that the antiprismatic graphs are exactly the graphs in Free$\{K_{1,3}, 2P_1 + P_2, 4K_1\}$. Therefore, solving the vertex coloring problem in that class is equivalent to solving the clique covering problem in prismatic graphs.

Chudnovsky and Seymour gave a full structural description of prismatic graphs. They divided the class in two subclasses : the orientable prismatic graphs and the non-orientable prismatic graphs. Preissman, Robin and Trotignon gave an algorithm to solve the clique covering problem in non-orientable prismatic graphs in polynomial time.

We show that this algorithm can also be used to solve the problem on prismatic graphs with no induced $2K_1 + C_4$, whether they are orientable or not. To achieve this, we show that each graph in this class has a bounded number of vertex disjoint triangles through a cautious analysis of their structure.

Références

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