

A finite presentation of graphs of treewidth at most three and associated connectivity decompositions

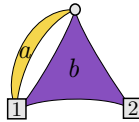
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We have recently shown the existence of a finite equational presentation of graphs of treewidth at most three [1], solving an instance of an open problem by Courcelle and Engelfriet [2, p. 118].

We use a syntax generalising series-parallel expressions, denoting graphs with a small interface (squared vertices below) :

$$t, u ::= t \parallel u \mid \mathbf{1}t \mid \mathbf{f}t \mid pt \mid a \mid \emptyset$$

For example the term $\mathbf{f}(b \parallel (32)\mathbf{1}a)$ denotes the following graph :



We introduce appropriate notions of connectivity for such graphs (components, cutvertices, separation pairs). We use those concepts to analyse the structure of graphs of treewidth at most three, showing how they can be decomposed recursively, first canonically into connected parallel components, and then non-deterministically. The main difficulty consists in showing that all non-deterministic choices can be related using only finitely many equational axioms.

Références

- [1] A. Doumane, S. Humeau, and D. Pous, *A Finite Presentation of Graphs of Treewidth at Most Three*. In 51st International Colloquium on Automata, Languages, and Programming (ICALP 2024). Leibniz International Proceedings in Informatics (LIPIcs), Volume 297, pp. 135 :1-135 :18, Schloss Dagstuhl – Leibniz-Zentrum für Informatik (2024) <https://doi.org/10.4230/LIPIcs.ICALP.2024.135>
- [2] Bruno Courcelle and Joost Engelfriet *Graph Structure and Monadic Second-Order Logic - A Language-Theoretic Approach*, Encyclopedia of mathematics and its applications **138**, Cambridge University Press (2012).